CSU High School Mathematics Tournament
March 6^{th}, 2004

Ciphering Competition

ROUND 1, QUESTION 1

SCHOOL NAME: ___
SCHOOL NUMBER:

Find the real solutions x of the equation

$$\frac{1}{1+\dfrac{1}{1+\dfrac{1}{x}}} = x \ .$$

ANSWER: $\pm\dfrac{\sqrt{2}}{2}$

ROUND 1, QUESTION 2

SCHOOL NAME: ___
SCHOOL NUMBER:

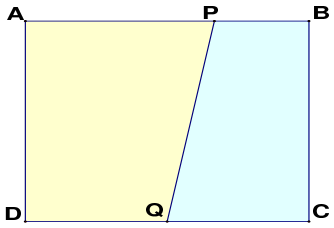
If $\sin(\alpha) = \frac{1}{3}$ **what is the numerical value of** $\cos(2\alpha)$**?**

ANSWER: $\frac{7}{9}$

ROUND 1, QUESTION 3

SCHOOL NAME: ___
SCHOOL NUMBER:

In the square $ABCD$ the point P on $\overline{AB}$ satisfies $AP = 2PB$ and the point Q is the midpoint of the side $\overline{CD}$.
Find the ratio $\frac{Area(APQD)}{Area(PBCQ)}$.



ANSWER: $\frac{7}{5}$

ROUND 1, QUESTION 4

SCHOOL NAME: ___
SCHOOL NUMBER:

One picks two whole numbers at random. What is the probability that their product is even?

ANSWER: $\frac{7}{8}$

ROUND 1, QUESTION 5

SCHOOL NAME: ___
SCHOOL NUMBER:

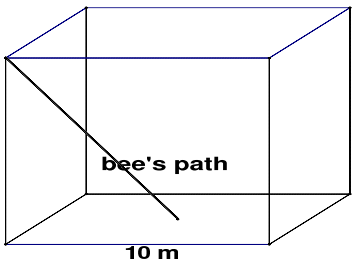
Two positive integers x and y satisfy $x + y = xy$. What is $x + y$?

ANSWER: 4

ROUND 1, QUESTION 6

SCHOOL NAME: ___
SCHOOL NUMBER:

A bee flies inside of a cubic box with side lengths 10 meters from one corner directly to the center of a non-adjacent face. How many meters does it fly?



ANSWER: $5\sqrt{6}$ **or** $\sqrt{120}$

ROUND 1, QUESTION 7

SCHOOL NAME: ___
SCHOOL NUMBER:

Solve for real x:

$$2^{2x} + 2^x = 30 \ .$$

ANSWER: $\dfrac{\ln 5}{\ln 2}$ **or** $\log_2 5$

ROUND 1, QUESTION 8

SCHOOL NAME: ___
SCHOOL NUMBER:

Determine if the following statement is true or false: "For all nonzero integers x and y either $2x = 3y + 7$ or $x^2 + y^2 \geq 2$".

ANSWER: true

ROUND 2, QUESTION 1

SCHOOL NAME: ___
SCHOOL NUMBER:

Solve the equation

$$1 + x[1 + x(1 + x)] = \frac{1}{x - 1} \ .$$

ANSWER: $\pm\sqrt[4]{2}$ **and** $\pm\sqrt[4]{2}i$

ROUND 2, QUESTION 2

SCHOOL NAME: ___
SCHOOL NUMBER:

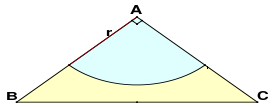
What is the smallest integer value of n such that $n \cos\left(\frac{\pi}{6}\right) > 30$? (angles are measured in radians)

ANSWER: 35

ROUND 2, QUESTION 3

SCHOOL NAME: ___
SCHOOL NUMBER:

The area of an isosceles right triangle $\triangle ABC$ ($m\angle BAC = 90°$) is divided into two equal parts by an arc of a circle centered at A and of radius r. Find r in terms of AB.



ANSWER: $\dfrac{AB}{\sqrt{\pi}}$

ROUND 2, QUESTION 4

SCHOOL NAME: ___
SCHOOL NUMBER:

A whole number is chosen at random from the set $\{1, 2, 3, ..., 2004\}$. What are the chances that the number is divisible by 167?

ANSWER: $\frac{1}{167}$

ROUND 2, QUESTION 5

SCHOOL NAME: ___
SCHOOL NUMBER:

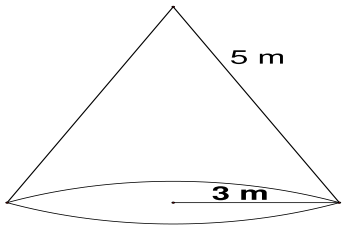
Two positive integers x and y satisfy the equation $x^2 - xy = 2$. What is the value of y?

ANSWER: 1

ROUND 2, QUESTION 6

SCHOOL NAME: ___
SCHOOL NUMBER:

What is the volume (in cubic meters) of the right circular cone in the figure below whose base radius is 3 meters and slide height is 5 meters?



ANSWER: 12π

ROUND 2, QUESTION 7

SCHOOL NAME: ___
SCHOOL NUMBER:

Solve for the positive x the logarithmic equation

$$\ln x + \ln(x + 20) = \ln 3 + 2 \ln 10.$$

ANSWER: 10

ROUND 2, QUESTION 8

SCHOOL NAME: ___
SCHOOL NUMBER:

Given $M = (1+\sqrt{2})^{2004}+(1-\sqrt{2})^{2004}$ your answer to this problem should be A if M is a whole number or B if M is an irrational number. What is your answer?

ANSWER: A

Backup Problem 1

SCHOOL NAME: ___
SCHOOL NUMBER:

Solve the system of equations:

$$\begin{cases} 2x + y + z = 3 \\ x + 2y + z = 7 \\ x + y + 2z = 6 \end{cases}$$

ANSWER: $x = -1, y = 3, z = 2$

Backup Problem 2

SCHOOL NAME: ___
SCHOOL NUMBER:

How many different ordered pairs (x, y) of real numbers satisfy the equation

$$x^2 + y^2 + 9 = xy + 3x + 3y \ ?$$

ANSWER: One

Backup Problem 3

SCHOOL NAME: ___
SCHOOL NUMBER:

A fair coin is to be tossed 4 times. What is the probability that more of the tosses will result in heads than will result in tails?

ANSWER: 5/16