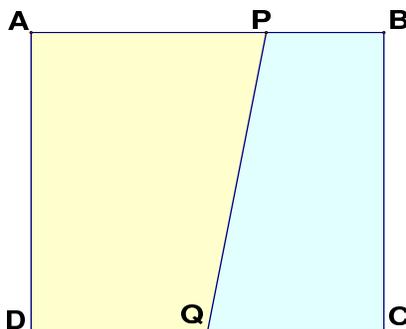1. Find the real solutions x of the equation

$$\frac{1}{1 + \frac{1}{1 + \frac{1}{x}}} = x \ .$$

2. If $\sin(\alpha) = \frac{1}{3}$ what is the numerical value of $\cos(2\alpha)$?

3. In the square $ABCD$ the point P on $\overline{AB}$ satisfies $AP = 2PB$ and the point Q is the midpoint of the side $\overline{CD}$. Find the ratio $\frac{Area(APQD)}{Area(PBCQ)}$.

4. One picks two whole numbers at random. What is the probability that their product is even?

5. Two positive integers x and y satisfy $x + y = xy$. What is $x + y$?

6. A bee flies inside of a cubic box with side lengths 10 meters from one corner directly to the center of a non-adjacent face. How far does it fly?

bee's path

10 m
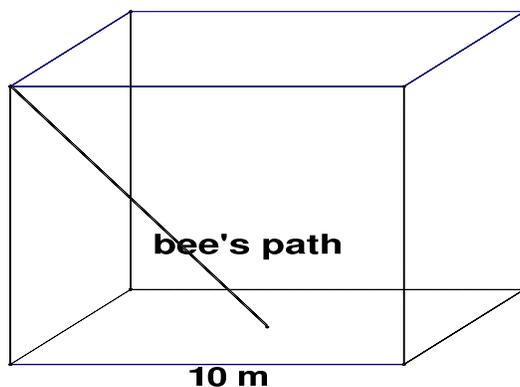
7. Solve for real x:
$$2^{2x} + 2^x = 30 \ .$$

8. Determine if the following statement is true or false: "For every nonzero integers x and y either $2x = 3y + 7$ or $x^2 + y^2 \geq 2$".
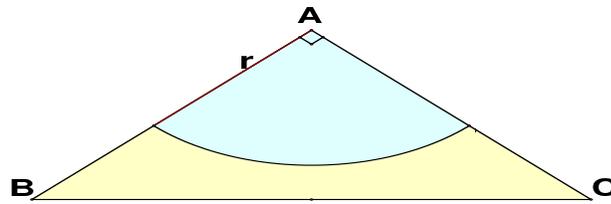
Round 2

1. Solve the equation
$$1 + x[1 + x(1 + x)] = \frac{1}{x - 1} \ .$$

2. What is the smallest integer value of n such that $n \cos\left(\dfrac{\pi}{6}\right) > 30$? (angles are measured in radians)

3. The area of an isosceles right triangle $\triangle ABC$ ($m\angle BAC = 90°$) is divided into two equal parts by an arc of a circle centered at A and of radius r. Find r in terms of AB.

4. A whole number is chosen at random from the set $\{1, 2, 3, ..., 2004\}$. What are the chances that the number is divisible by 167?

5. Two positive integers x and y satisfy the equation $x^2 - xy = 2$. What is the value of y?

6. What is the volume (in cubic meters) of a right circular cone in the figure below whose base radius is 3 meters and slide height is 5 meters?
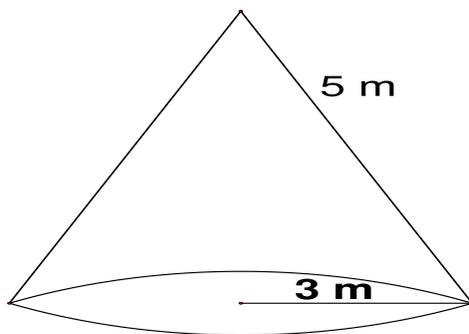
7. Solve for the positive x the logarithmic equatiion

$$\ln x + \ln(x + 20) = \ln 3 + 2 \ln 10.$$

8. Given $M = (1+\sqrt{2})^{2004} + (1-\sqrt{2})^{2004}$ your answer to this problem should be A if M is a whole number or B if M is an irrational number. What is your answer?

Backup problems

1. Solve the system of equations:

$$\begin{cases} 2x + y + z = 3 \\ x + 2y + z = 7 \\ x + y + 2z = 6 \end{cases}$$

2. How many different ordered pairs (x, y) of real numbers satisfy the equation

$$x^2 + y^2 + 9 = xy + 3x + 3y \ ?$$

3. A fair coin is to be tossed 4 times. What is the probability that more of the tosses will result in heads than will result in tails?