Mathematics Tournament 2013 Ciphering

Sponsored by

The Columbus State University

Department of Mathematics and Philosophy

March 2nd, 2013

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Do not open your test until instructed to do so!

Round One

1. If $(x+1)(x-1) = 8$, determine the numerical value of $(x^2+x)(x^2-x)$.

   Ans:

2. The function $g(x)$ satisfies the equation $g(x) = g(x+1) + g(x-1)$ for all values of x. If $g(1) = 1$ and $g(2) = 3$, what is the value of $g(7)$?

   Ans:

3. A carpenter wishes to construct a ladder with nine rungs whose lengths decrease uniformly from 24 inches at the base to 18 inches at the top. What is the length of the rung fouth from the top?

   Ans:

4. A triangle has side-lengths equal to $8\sqrt{33}$, $15\sqrt{33}$ and $17\sqrt{33}$. What is its area?

   Ans:

5. Find the value of
   $$\log_2 3 \cdot \log_3 4 \cdot ... \cdot \log_{2012} 2013 \cdot \log_{2013} 2.$$

   Ans:

6. Compute the exact value of
   $$(\sin 22.5° - \cos 22.5°)^2$$

   Ans:

7. Suppose the functions $f(x)$ and $g(x)$ satisfy the system of equations
$f(x) + 3g(x) = x^2 + x + 6$
$2f(x) + 4g(x) = 2x^2 + 4$
for all x. Find $g(x)$.

Ans:

8. If $ac + ad + bc + bd = 102$ and $c + d = 6$, what is the value of $a + b + c + d$?

Ans:

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Round Two

1. Two consecutive positive integers x and y satisfy the equation $x^2 - xy = 6$. What is the value of $x + y$?

   Ans:

2. In a group of 20 students taking Sciences, 6 of them are taking both Chemistry and Biology, 9 of them are taking Chemistry and 12 students are taking Biology. What is the percentage of students taking neither Chemistry nor Biology ?

   Ans:

3. If $f(x) = px + q$ and $f(f(x)) = 4x + 12$, where p and q are positive real numbers, what is the value of q?

   Ans:

4. Given angles A and B such that,

$$\sin^2 A + \cos^2 B = \frac{3}{2}x$$

$$\cos^2 A + \sin^2 B = \frac{1}{2}x$$

   Determine the value of x.

   Ans:

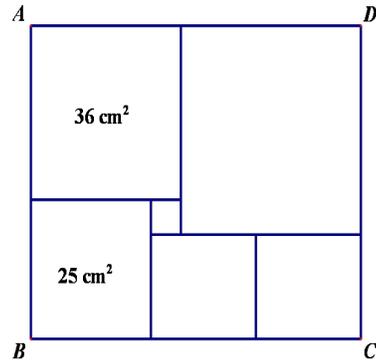5. Evaluate $\frac{1}{8} + \log_2 2\sqrt{2\sqrt{2}}$.

   Ans:

6. Let α be a solution of $x^2 + x + 1 = 0$. What is the value of

$$\alpha^6 + \alpha^5 + \alpha^4 + \alpha^3 + \alpha^2 + \alpha + 1?$$

Ans:

7. Rectangle $ABCD$ is made up of six squares. The areas of two of the squares are shown. Find the perimeter of rectangle $ABCD$, in centimeters.

Ans:

8. If $S = 3^{2013} + 3^{-2012}$ and $T = 3^{2013} - 3^{-2012}$, what is the exact value of $S^2 - T^2$?

Ans: