

ROUND I

1) Find all real solutions of the equation $\sqrt{10-x} - x - 2 = 0$. Answer $x = 1$

2) Ten percent of the students taking a math exam fail the exam. Forty percent of the students taking the exam are boys and fifteen percent of these boys fail the exam. Twenty girls fail the exam. How many total students took the exam? Answer: 500

3) If a, b are the roots of the equation $x^2 + x - 1 = 0$, find $a^2 + b^2$. Answer 3

4) If the expression $(3 + x - 2x^2)^5$ is fully expanded and simplified, what is the sum of the coefficients? Answer: 32

5) How many rectangles are there in this figure?



Answer: 36

6) Suppose θ is an acute angle such that $\cos 2\theta = \frac{1}{3}$. What is the value of $\sin^2 \theta \cos^2 \theta$?

Answer: 2/9

7) How many different 3-digit numbers can be written using no 0's and at least one 5? Answer: 217

8) The new family across the street has 2 children, at least one of which is a boy. What is the probability that both are boys? Answer: 1/3

ROUND II

- 1) Find the sum of all the solutions to the equation $|2x^2 - x - 3| = 5 - x$. The solutions are $-2, 2, \frac{1}{2} + \frac{\sqrt{3}}{2}i$ and $\frac{1}{2} - \frac{\sqrt{3}}{2}i$ thus, Answer: 1
- 2) A number is randomly picked from $1, 2, 3, \dots, 20$. What is the probability that the number is a perfect square or a perfect cube? Answer: $\frac{1}{4}$
- 3) Ben takes 2 hours to wash 500 dishes, and Tim takes 3 hours to wash 450 dishes. How long will they take, working together, to wash 1000 dishes? Answer: 2.5 hours
- 4) Find the sum of all the prime numbers that leave the same remainder when divided into each of the numbers 279 and 357. Ans. 18
- 5) The number bacteria in a bottle doubles every minute. After 60 minutes the bottle is full of bacteria. How long did it take to fill half the bottle? Ans. 59 minutes
- 6) Find the area of the bounded region defined by the graph of the equation $|x| + |y| = 4$.
Answer 32
- 7) The product of 2^{21} and 5^{17} is an integer. How many digits does it have? Answer: 19
- 8) A pile of gold dust is divided among three prospectors. Jane gets $\frac{2}{5}$ of the dust, Lena gets $\frac{1}{4}$ of the dust, and Billy gets the remaining 14 grams. How many grams does Lena get?
Answer 40 grams