Math Tournament 2014 – Ciphering

Round 1

1) Find the value of $(-1)^0 + (-1)^1 + (-1)^2 + \cdots + (-1)^{2014}$.
Answer : 1

2) Tim lights up a candle every 10 minutes. Every candle burns for exactly 40 minutes. How many candles are burning exactly 55 minutes after Tim lit up the first candle?
Answer : 4

3) What is the sum of the digits of the number x that satisfies

$$\frac{1}{2} + \frac{1}{19} + \frac{1}{53} = \frac{x}{2014}?$$

Answer : 8

4) Jim listed his house for sale in December. In January he increased the price by 10%, but in February he lowered the price by 10%. Find the percentage change in the initial value of the house.
Answer : $\pm 1\%$

5) If a and b are solutions of the equation $x^2 + 2013x - 2014 = 0$ then what is the value of $\frac{1}{a} + \frac{1}{b}$?
Answer : $\frac{2013}{2014}$

6) The sides of a trapezoid are positive integers. If its perimeter is 5 then what is the measure, in degrees, of the smallest angle?
Answer : 60°

7) For how many positive integers n is the number $n^2 - 3n + 2$ prime?
Answer : One.

8) If you write the numbers from 1 to 100 then how many times will you write the digit 7?
Answer : 20

Round 2

1) Find the value of $1 - 2 + 3 - 4 + \cdots + 2013 - 2014$.
Answer : -1007

2) The number 34 is written on a screen. After each minute the screen changes and displays a new number obtained by adding 18 to the product of the digits of the old number. What number is displayed during the third minute?
Answer : 18

3) What is the smallest 3-digit prime number?
Answer : 101

4) Positive integers a, b, and c satisfy the relations $a \cdot b = 15$, $b \cdot c = 21$, and $a \cdot c = 35$. What is the value of $a \cdot b \cdot c$?
Answer : 105

5) In the Atlantic Ocean the ratio of the mass of the salt in the water to the mass of unsalted water is $6 : 194$. How many kilograms of salt are in 1000 kilograms of sea water?
Answer : 30

6) Solve the equation $8^{4x} = 16^{2x-1}$.
Answer : -1

7) Denote by S the number of perfect squares from 1 to 2014^6 and by Q the number of perfect cubes from 1 to 2014^6. Find the value of $\dfrac{S}{Q}$.
Answer : 2014

8) Find the product of all positive integers which are factors of 12.
Answer : 1728

Backup Questions

1) How many seconds are in a 24-hour day?
Answer : $86,400$

2) Solve the equation in complex numbers

$$\frac{1}{(x+i)} + \frac{1}{(1+(x+1)i)} = \frac{1}{(x-1+i)}.$$

Answer : 2 and $(-1-i)$.