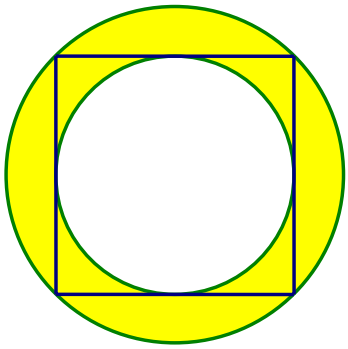
ROUND I

1. Brian has a pair of numbers. The cube root of their difference is the smallest odd prime. The square root of their sum is the smallest odd perfect square greater than 1. What are the numbers?

2. On the first day of classes the ratio of boys to girls enrolled in Dr. Henning's class was $5/7$. On the second day, 4 boys and 4 girls dropped the class and the ratio changed to $2/3$. How many boys were enrolled on the first day?

3. Solve the equation $\sqrt{x^2 - 2x + 1} = 2x$.

4. Given a square, construct a circumscribed circle and an inscribed circle as indicated in the figure. If the area of the ring formed by the two circles (shaded in the diagram) is 4, find the radius of the outer circle.



5. A certain ball, when dropped from any height, moves always in a vertical path and bounces to 1/3 of its original height. If the ball is dropped from 54 ft, what is the total distance that the ball has traveled when it hits the ground for the fourth time?

6. A number is selected at random from the set $\{100, 101, 102, \ldots, 999\}$. What is the probability that the selected number does **not** contain any of the digits 3, 5, 6, or 7?

7. Let $f(x) = (x+2)^2 - 1$ and $g(x) = -2 + \sqrt{x+1}$ for $x \geq -1$. Give a simplified form for the expression of $g(f(x))$ for all values of x such that $x \leq -2$.

8. Ron drives from home to school at a constant speed of 45 mph. If he wants the average speed of the round trip to be 40 mph, at what constant speed must he drive from school to home?

ROUND II

1. A 45 year old man has two children. One of the children is 3 years older than the other. Seven years from now, the father will have the same age as the sum of the ages of the children. What is the age of the younger child now?

2. Three runners enter a race. If runner A is 3 times as likely to win as runner B, and runner B is 4 times as likely to win as runner C, what is the probability that runner A wins the race?

3. Find the sum of the terms of the finite sequence $9,\ 18,\ 27,\ldots,\ 1998,\ 2007$.

4. Find the solution set of $x^2 - 8x - 36 \le x \le x^2 - 2x - 108$.

5. Cindy has a rectangular garden that is 16 feet long and 8 feet wide. She wants to put a strip of gravel of uniform width around the outside of the garden. She has enough gravel to cover 112 square feet. How wide should she make the strip?

6. How many times will the graph of $|x| + |y| = 2$ intersect the graph of $x^2 - y = 2$?

7. The center of a circle of radius $\frac{1}{2}$ is selected at random on the xy-plane. What is the probability that the circle will enclose a point with integer coordinates?

8. As shown in the figure, a regular hexagon is constructed by joining some of the midpoints on the edges of a cube. The sides of the cube have length 2. Find the area of the hexagon.