

1. Which of the following intervals contains **all** of the real zeros of the function

$$g(t) = 2t^3 - 3t^2 + t?$$

- (a) $(-1, 2)$ (b) $(\frac{1}{4}, 2)$ (c) $(-1, \frac{1}{4})$ (d) $(1, 2)$ (e) $(-2, 1)$

2. The expression $((2a)^{-1} + b)^{-1}$ is equivalent to which of the following expressions?

- (a) $\frac{1 + 2ab}{2a}$ (b) $\frac{2ab + 1}{b}$ (c) $\frac{2a}{1 + 2ab}$ (d) $\frac{b}{2ab + 1}$ (e) $\frac{2a}{b}$

3. Find the **sum** of the solutions of the equation $\frac{2}{x} = \frac{3}{x - 2} - 1$.

- (a) 4 (b) -1 (c) 0 (d) 3 (e) 2

4. Simplify the expression $1 + i + i^2 + i^3 + \dots + i^{16}$, where $i = \sqrt{-1}$.

- (a) 0 (b) 1 (c) -1 (d) i (e) $-i$

5. If $\log_{b^2} x + \log_{x^2} b = 1$, $b > 1$, and $x \neq 1$ then x equals what value?

- (a) b^2 (b) $b + 2$ (c) $\frac{1}{2}$ (d) b (e) \sqrt{b}

6. Find the area (in square inches) of a circle inscribed in a rhombus whose perimeter is 100 inches and whose longer diagonal is 40 inches.

- (a) 144π (b) $\frac{144\pi}{25}$ (c) 12π (d) $\frac{12\pi}{5}$ (e) 225π

7. The polynomial $P(x) = x^5 - 9x^4 + 29x^3 - 45x^2 + 54x - 54$ can be written in the form $P(x) = (x - 3)^n Q(x)$, where n is a positive integer and $Q(x)$ is not divisible by $x - 3$. What is n ?

- (a) 4 (b) 1 (c) 2 (d) 3 (e) 5

8. Given that $p(x) = 16x^2 + ax - 33$, find the value of a so that $p(-\frac{3}{8}) = p(\frac{11}{2}) = 0$.

- (a) -451 (b) -82 (c) 484 (d) 100 (e) -90

9. Determine the **number** of distinct solutions of the equation $4x^2 - 68x + 289 = 0$.

- (a) 0 (b) 2 (c) 3 (d) 4 (e) 1

10. The continued fraction

$$1 + \frac{1}{x + \frac{3}{x+7}}$$

can be expressed as $\frac{x^2 + ax + b}{x^2 + cx + d}$. Find $a + b + c + d$.

- (a) 28 (b) 12 (c) 30 (d) 8 (e) 19

11. How many subsets of $\{ 1, 2, 3, 4, 5, 6, 7, 8, 9, 10 \}$ contain at least one odd integer?

- (a) 32 (b) 100 (c) 1024 (d) 992 (e) 31

12. Find the value for x such that $4 = \log_x \frac{1}{16}$.

- (a) 2 (b) 4 (c) $\frac{1}{2}$ (d) -2 (e) 16

13. Find the radius of the circle with center $(2, 4)$ that is tangent to the line $x + y - 4 = 0$.

- (a) $\sqrt{2}$ (b) $\sqrt{3}$ (c) $\frac{\sqrt{5}}{2}$ (d) $\frac{2\sqrt{3}}{3}$ (e) $\frac{2\sqrt{7}}{3}$

14. Provided that $c \neq 0$, define $[a, b, c]$ to mean $\frac{a+b}{2c}$. What is the value of $[[8, 24, 2], [48, 0, 1], [100, 200, 75]]$?

- (a) 16 (b) 24 (c) 2 (d) 10 (e) 8

15. Ann, Bob, Carl and Doc are all thinking of the same natural number. Ann says it consists of two digits. Bob says it is a divisor of 150. Carl says it is not 150. Doc says it is divisible by 25. Given that exactly three of them are telling the truth, which of them is not?

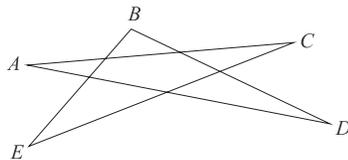
- (a) Doc (b) Ann (c) Bob
(d) Carl (e) cannot be determined

16. Find the sum of the solutions of $|x - 5| + 3 = |2x + 5|$.

- (a) -19 (b) -12 (c) -21 (d) -13 (e) 1

17. (*) Referring to the figure below, if S denotes the sum of the angles $A, B, C, D,$ and E (measured in radians), then what can be said about the value of S ?

- (a) $S < \pi$ (b) $S = \pi$ (c) $S = \frac{5\pi}{3}$
(d) $S = \frac{3\pi}{2}$ (e) $S > \frac{5\pi}{3}$



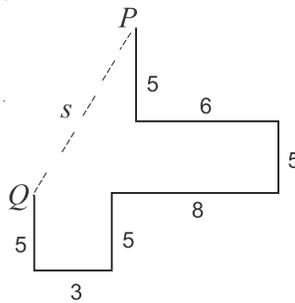
18. What is the area of the largest triangle that can be inscribed within a circle of radius r , given that one of the triangle's sides must coincide with a diameter of the circle?
- (a) $2r^2$ (b) $\frac{1}{2}r^2$ (c) r^2 (d) $\frac{\sqrt{3}r}{2}$ (e) $2r$
19. Suppose that a 1-inch slice is to be removed from a cube. If taking a slice parallel to a face leaves 648 cubic inches, what is the side length (in inches) of the cube?
- (a) $\sqrt[3]{900}$ (b) $\sqrt{898}$ (c) 11 (d) 10 (e) 9
20. If a jet plane flies at an altitude of 5 miles above the earth's equator and circles the globe once, how much farther (in miles) than the circumference of the equator does it fly?
- (a) 4π (b) 10π (c) 14π (d) 25π (e) 8π
21. Calculate the fifth root of $5^{(5^5)}$.
- (a) 5^5 (b) $5^{(5^5-1)}$ (c) $5^{(4^5)}$ (d) $5^{(5^4)}$ (e) $(\sqrt{5})^5$
22. (*) Let V denote the set of natural numbers, n , such that n is the sum of the squares of three consecutive positive integers. Which of the following is true?
- (a) All elements of V are even.
(b) No element of V is divisible by 3.
(c) All elements of V are odd.
(d) No element of V is divisible by 5.
(e) No element of V is divisible by 11.

23. Find the circumference of a circle circumscribed about a square having side length 8.

- (a) $8\pi\sqrt{2}$ (b) 8π (c) 32
(d) 64π (e) none of these

24. The following path from P to Q consists of vertical and horizontal straight line segments having lengths as shown. If s is the diagonal distance from P to Q , find s .

- (a) $\sqrt{214}$ (b) $\sqrt{255}$ (c) $3\sqrt{15}$ (d) $5\sqrt{5}$ (e) $4\sqrt{15}$



25. If $f(x) = \sqrt{x} + \sqrt{x+2}$, compute $\frac{1}{f(1)} + \frac{1}{f(3)} + \frac{1}{f(5)}$.

- (a) 1 (b) $\frac{\sqrt{7}-1}{2}$ (c) $\frac{\sqrt{7}+3}{2}$ (d) $\frac{\sqrt{7}+2}{2}$ (e) $\frac{\sqrt{7}+1}{2}$

26. A package of six light bulbs contains two defective bulbs. If three bulbs are to be selected at random for use, find the probability that none of the three are defective.

- (a) $\frac{2!}{6!}$ (b) $\frac{3!}{6!}$ (c) $\frac{3! \cdot 2!}{6!}$ (d) $\frac{4!}{6!}$ (e) $\frac{4! \cdot 3!}{6!}$

27. Suppose B and C are points on straight line segment AD (pictured below), with the property that $\overline{AB} = \overline{BC} = \overline{CD}$. What percent of \overline{AC} is \overline{AD} ?

- (a) $133\frac{1}{3}\%$ (b) 150% (c) $33\frac{1}{3}\%$ (d) 50% (e) $66\frac{2}{3}\%$



28. If 60% of x is 40% of y , and y is 30% of z , then x is what percent of z ?

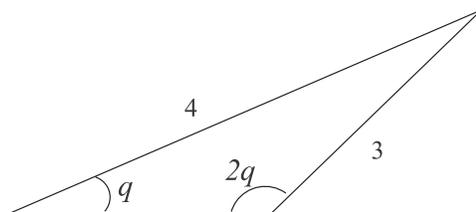
- (a) 30% (b) 110% (c) 50% (d) 20% (e) 72%

29. Compute $\left(\frac{1+i\sqrt{3}}{2}\right)^{100}$.

- (a) $\frac{1+i\sqrt{3}}{2}$ (b) $\frac{-1+i\sqrt{3}}{2}$ (c) $\frac{-1-i\sqrt{3}}{2}$
 (d) -1 (e) $\frac{1-i\sqrt{3}}{2}$

30. Referring to the figure below, determine the measure of q in degrees.

- (a) 30° (b) 48.2° (c) 38.6° (d) 50.1° (e) 49.7°



31. The area of a right triangle is 20 square inches. The ratio of its legs is 2:5. Find the number of inches in the hypotenuse of this triangle.

- (a) $\sqrt{29}$ (b) $\sqrt{58}$ (c) $4\sqrt{29}$ (d) $2\sqrt{29}$ (e) $5\sqrt{29}$

32. A college graduate goes to work for x dollars per week. After several months the company gives all employees a 20% pay cut. A few months later the company gives all employees a 20% raise. What is the college graduate's new weekly salary?

- (a) $0.96x$ (b) $0.8x$ (c) x (d) $1.02x$ (e) $1.2x$

33. In the sequence of numbers 1,3,2,-1,... each term after the first two terms is equal to the term preceding it minus the term preceding that one. Determine the sum of the first one hundred terms of the sequence.

- (a) 4 (b) 5 (c) 2 (d) 1 (e) -1

34. Find the sum of the digits of the number $(10^{3n^3+9} + 1)^2$, where n is a positive integer.

- (a) $3n$ (b) 1 (c) $3n^3$ (d) 4 (e) n^3+n+3

35. Let $a_0 = 1$ and $b_0 = 2$. For all $n \geq 1$, let

$$a_n = 2a_{n-1} + 3b_{n-1}, \text{ and}$$
$$b_n = a_{n-1} - 2b_{n-1}.$$

What is the remainder when a_3 is divided by 4?

- (a) 3 (b) 1 (c) 2
(d) 0 (e) none of these

41. Find the area (in square centimeters) of a circular sector having a 30° central angle and a 6 cm radius.

- (a) 3π (b) $\frac{9\sqrt{3}}{2}$ (c) 6π
(d) π (e) none of these

42. A positive integer N with three digits in its base ten representation is chosen at random. If each three digit number has an equal chance of being selected, determine the probability that $\log_2 N$ is an integer.

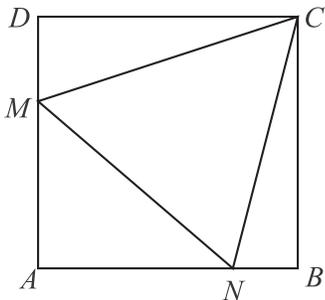
- (a) 0 (b) $\frac{3}{899}$ (c) $\frac{1}{225}$ (d) $\frac{1}{450}$ (e) $\frac{1}{300}$

43. An assembly operation for a computer circuit board consists of four distinct operations, which can be performed in any order. One of the operations involves soldering a wire to a microchip. If all possible assembly orderings are equally likely, what is the probability that the soldering operation comes first or second?

- (a) $\frac{1}{3}$ (b) $\frac{1}{4}$ (c) $\frac{1}{6}$ (d) $\frac{1}{8}$ (e) $\frac{1}{2}$

44. (*) In the figure below $ABCD$ is a square and CMN is an equilateral triangle. If the area of $ABCD$ is one square inch, find the area of CMN in square inches.

- (a) $1 - \sqrt{3}/3$ (b) $\sqrt{3}/4$ (c) $2\sqrt{3} - 3$ (d) $\sqrt{2}/3$ (e) $4 - 2\sqrt{3}$



45. Assume a and b are nonzero real numbers. What is the maximum of the function $f(x) = a \sin x + b \cos x$?
- (a) $|a + b|$ (b) $\max\{|a|, |b|\}$ (c) $\sqrt{a^2 + b^2}$
- (d) $|a| + |b|$ (e) $\frac{\sqrt{2}}{2}(|a| + |b|)$
46. (*) For each natural number n , let $s_n = 1 - 2 + 3 - 4 + 5 - 6 + \dots + (-1)^{n-1}n$. What can be said about the value of $s_{1999} + s_{2000}$?
- (a) negative (b) 1 (c) 2
- (d) 0 (e) greater than 2
47. Find the smallest positive integer m such that the graph of $y = mx$ intersects the graph of $y = \frac{5}{4}x^2 + 44$.
- (a) 14 (b) 8 (c) 15 (d) 1 (e) 17
48. The lengths of two sides of a triangle are 7 and 10. Between what two numbers does the length of the third side lie?
- (a) 3 and 17 (b) 0 and 3 (c) 17 and 70
- (d) 17 and 20 (e) none of these
49. The first three terms of a geometric sequence are $2x - 4$, $5x - 7$ and $10x + 4$ for some value of x . What is the fourth term of the sequence?
- (a) 5 or $\frac{13}{5}$ (b) -7 (c) -90
- (d) -5 or $-\frac{13}{5}$ (e) 150 or 162

50. Find a formula, in the variable x , for the distance between an arbitrary point (x,y) on the parabola $y = x^2$ and the point $(7, 7)$.

(a) $\sqrt{x^2 - 7}$

(b) $\sqrt{x^4 - 13x^2 - 14x + 98}$

(c) $\sqrt{x^4 + 13x^2 + 14x + 98}$

(d) $\sqrt{x^4 + x^2 - 98}$

(e) $\sqrt{x^2 + x - 14}$